

# Nuclear diagnostics and Magnetic Resonance Imaging

## Lecture 6: Positron Emission Tomography II

K. Long  
Imperial College London/STFC

[K.Long@Imperial.ac.uk](mailto:K.Long@Imperial.ac.uk)

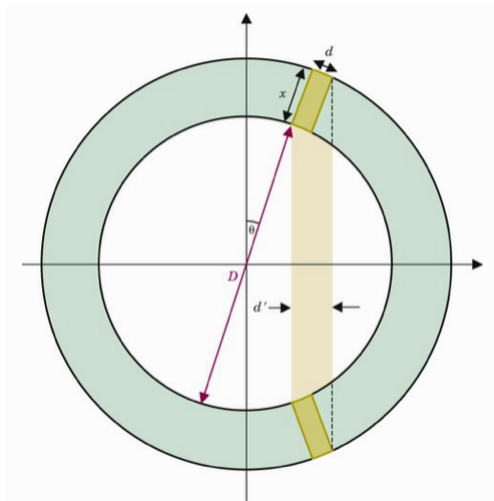
## 1 Positron Emission Tomography

- System resolution—continued
- Sensitivity
- Types of coincidence event
- Overview of PET systems
- “Data acquisition”
- Comparison of sensitivities
- Corrections
- Examples

## Section 1

# Positron Emission Tomography

## Reconstruction: depth-of-interaction (DOI) effect



Thickness of scintillator used to stop  $511 \text{ keV } \gamma$  introduces a reconstruction uncertainty

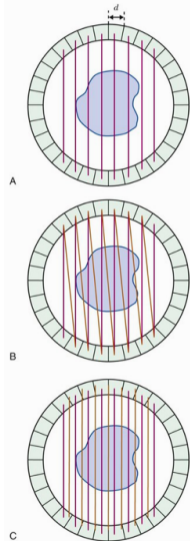
For the case sketched, the apparent width of the detector,  $d'$ , is given by:

$$d' = d \cos \theta + x \sin \theta$$

Can now use  $d'$  in the formulæ for, e.g.,  $R_{\text{int}}$

In a typical system, the DOI effect causes a degradation of  $\sim 40\%$  in the resolution at a distance of 100 mm from the centre

# Reconstruction: sampling effect



The intrinsic resolution is determined by the detector size,  $d$

- A) Sampling “frequency” determined by spacing, also  $d$   
Limits minimum feature size that can be resolved
- B) Record neighbouring coincidences  
Improved sampling; can reduce minimum feature size
- c) Treat “neighbouring coincidences” (B) as additional samples  
Implementation leads to improvement in detail in image

## Reconstruction: filter effect

Image reconstruction exploits techniques such as filtered back projection

Filters are used to suppress noise, but, removing frequencies from the Fourier transform of the image can also remove detail from the image

Image-processing strategies need to be tailored to the situation, e.g. brain scans may require different strategies to abdominal scans

# Sensitivity

Sensitivity is determined primarily by detector efficiency and solid angle coverage

True coincidence count rate  $\mathcal{R}_{\text{True}}$  for a positron-emitting source in air near midpoint between a pair of detectors is:

$$\mathcal{R}_{\text{True}} = \mathcal{R}_{e+} \epsilon^2 G \exp(-\mu T)$$

where:

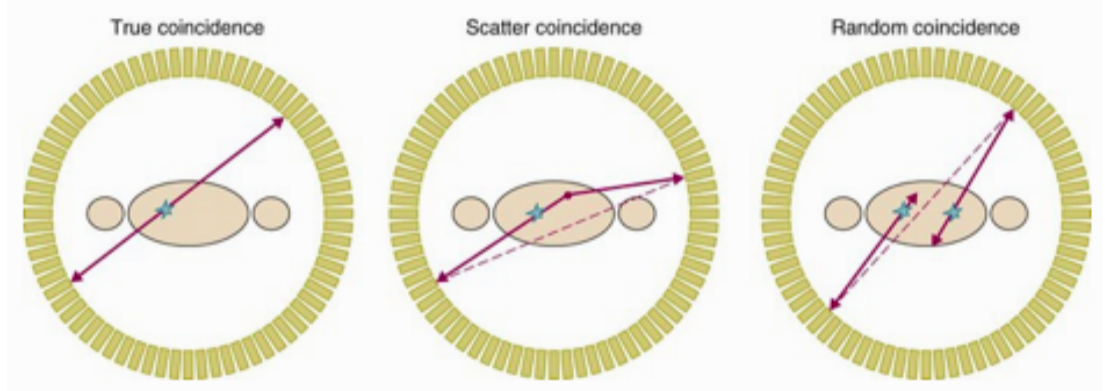
- $\mathcal{R}_{e+}$  is the rate of positron emission (positrons/sec)
- $\epsilon$  is the intrinsic detector efficiency  
(no of  $\gamma$ -rays recorded by detector)/(no of  $\gamma$ -rays 'hitting' detector)
- $G$  is the geometric efficiency of an individual detector  
$$G = \frac{2A_{\text{det}}}{3\pi D^2}$$
- $\mu$  is the linear attenuation coefficient,  $T$  the total thickness

# Sensitivity

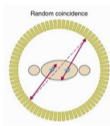
Intrinsic detector efficiency for a variety of scintillators

Scintillator	$\mu_{\text{scintillator}}$ ( $\text{cm}^{-1}$ )	$\mathcal{E}$ (2 cm)	$\mathcal{E}^2$ (2 cm)	Photon yield (per keV)
NaI (TI)	0.34	0.49	0.24	38
BGO	0.95	0.85	0.72	8
LSO	0.88	0.83	0.69	20-30
GSO	0.70	0.75	0.57	12-15

# Types of coincidence event



# Random coincidence



A random coincidence arises when two PMTs each receive a signal within the coincidence time window  $\Delta t = 2\tau$

“Singles rate” in detectors  $i$  and  $j$  are  $\mathcal{R}_{si}$  and  $\mathcal{R}_{sj}$ , random-coincidence rate:

$$\mathcal{R}_{\text{random}} = \Delta t \mathcal{R}_{si} \mathcal{R}_{sj}$$

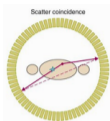
Ratio of random to true coincidences  $\propto \Delta t$ ; small  $\Delta t$  of benefit

$\mathcal{R}_{\text{random}}$  is proportional to the square of the activity in the body; true-coincidence rate rises linearly

Singles rates, and therefore  $\mathcal{R}_{\text{random}}$ , can be reduced with absorptive septa at the price of efficiency

Source of random coincidences not localised. Typical rates:  $\sim 0.1$  Hz (brain) to  $\sim 1$  Hz (abdomen). Reduces contrast & increases error in estimation of total dose

# Scatter coincidence



Scatter of one (or both) photons from single annihilation event in tissue or scanner

Pernicious since:

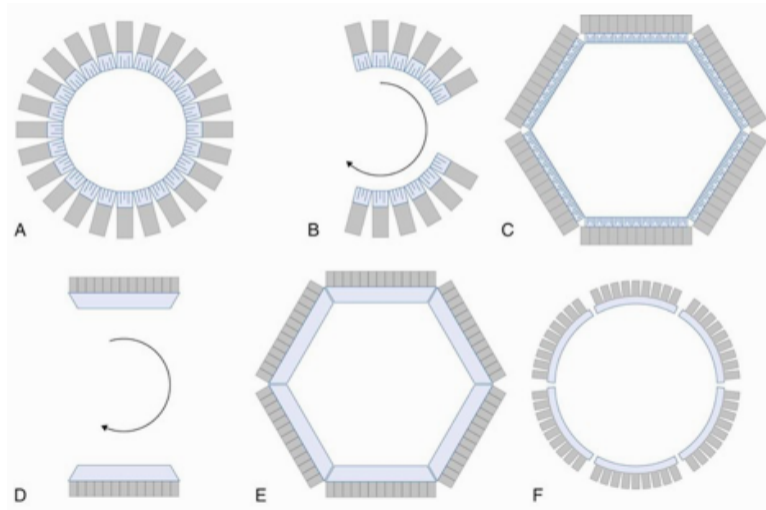
- Energy of scattered photon often differs little from decay photon  
→ falls within the energy detection window
- Scattered photon is in coincidence with unscattered photon  
tiny ( $\ll \Delta t$ ) additional delay from the increased path length

$\mathcal{R}_{\text{scatter}}$  proportional to activity in body; same dependence as true-coincidence rate

$\mathcal{R}_{\text{scatter}}$  can be reduced with absorptive septa at the price of efficiency

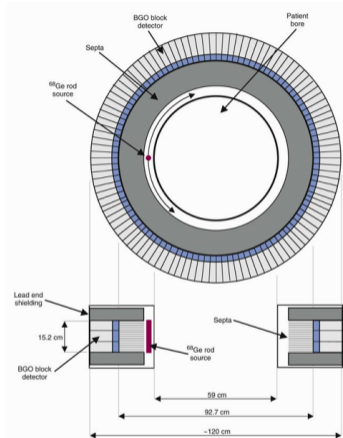
Typical rates: 0.2 – 0.5 Hz (brain) to 0.4 – 2 Hz (abdomen); scatter-event fraction can be as high as 60 – 70% for abdominal scans. Reduces contrast.

# Sample PET scanner geometries

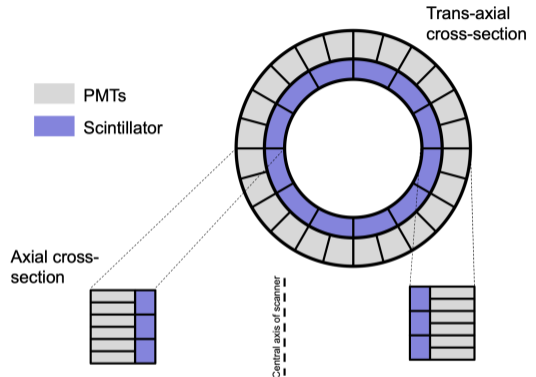


- A) Full ring of block detectors
- B) Partial ring of block detectors
- C) Hexagonal “ring” of quadrant-sharing panel detectors
- D) Dual-headed gamma camera
- E) Hexagonal array of gamma cameras
- F) Ring of curved plates of NaI(Tl)

# With and without septa

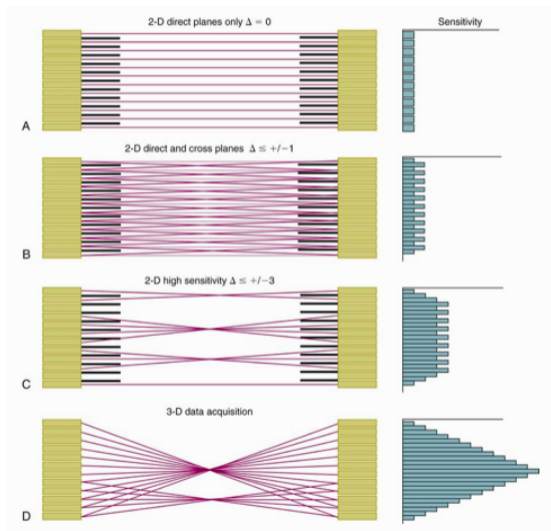


$^{68}\text{Ge}$  rod for transmission scans to allow absorption correction  
Retracted into Pb shield when not in use



Large increase in sensitivity  
Increase in ratio of  
scatter to true coincidences  
random to true coincidences

# Event recording topologies



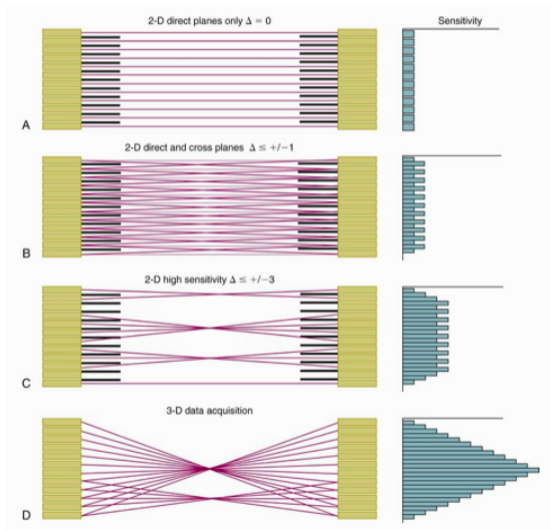
A) 2D data acquisition: septa such that:

- Accept only 'in line'  $\gamma$  pairs
- Reduced rate of scatters & randoms
- Recons: e.g. filtered back projection
- Least efficient

B) Cross planes: septa such that  $\Delta = \pm 1$ :

- Accept coincidences in neighbouring cells
- Increased rate of scatters & randoms
- Scanner centre, effective separation =  $\frac{d}{2}$
- Recons: as in A using effective separation
- For  $n$  rings  $\Rightarrow 2n - 1$  image planes
- Efficiency enhanced

# Event recording topologies



C) Cross planes: septa such that  $\Delta = \pm 2; \pm 3$ :

- Accept coins for cells with  $|\Delta| = 1, 2, 3$
- Increased rate of scatters & randoms
- Recons: for large  $|\Delta|$  resolution decreases
- Efficiency enhanced

D) 3D acquisition mode: no septa

- Accept all coincidences
- Sensitivity increases  $\times 4 - \times 8$
- Recons: require 3D algorithm
- Increased rate of scatters & randoms
- Most efficient

# Numerical comparison of sensitivities

Histograms in figures A–D indicate spacial dependence of relative efficiency;

→ indicates, for example, object of interest should be in the centre of a 3D scanner

For some 'typical' systems:

- PET scanner in 2D mode (1 slice per detector ring): 0.002–0.005 counts/s/Bq
- PET scanner in 3D mode (coincidences between all rings): 0.02–0.1 counts/s/Bq
- SPECT (for comparison): 0.0001–0.0003 counts/s/Bq

# Corrections; normalisation

Want response to be proportional to activity in a given pixel

PET systems have many (10,000–20,000) PMTs. Various factors determine the signal recorded for a given number of incident photons, including:

- Precise dimensions of crystal in neighbourhood of PMT;
- Coupling of crystal to PMT;
- $\gamma$ 's angle of incidence and crystal uniformity;
- PMT gain and gain in electronics;
- Response of CFD;

So, require to “normalise” signal

## Corrections; normalisation

Exploit rod source; take data while performing  $360^\circ$  rotation of rod

Normalisation factor,  $\mathcal{N}_{ij}$  for pair of PMTs  $i, j$  is given by:

$$\mathcal{N}_{ij} = \frac{N_{ij}}{\bar{N}}$$

where  $N_{ij}$  is the number of coincidences recorded between PMTs  $i$  and  $j$  and  $\bar{N}$  is the mean number of coincidences for all PMT pairs

Normalised number of coincidences,  $\mathcal{C}_{ij}$  for PMT pair  $i, j$  is given by:

$$\mathcal{C}_{ij} = \frac{C_{ij}}{\mathcal{N}_{ij}}$$

where  $C_{ij}$  is the number of coincidences recorded between PMTs  $i, j$  recorded in a patient scan

## Corrections; random coincidences; delayed window method

Coincidence time window  $\Delta t$  usually set to  $2\tau$ , where  $\tau$  is the width associated with the PMT timing signal

Take  $t_{\text{delay}} \gg \Delta t$ , then the number of coincidences between:

- Hit in PMT  $i$  at  $t_i$  and
- Hit in PMT  $j$  at  $t_j = t_i + t_{\text{delay}} \pm \Delta t$

will give an estimate of the random coincidence rate

The delayed-window estimate of the random coincidence rate between PMTs  $i$  and  $j$  can be subtracted from the measured coincidence rate

Subtraction of the random coincidence rate leads to an increase in the statistical (counting) uncertainty:

$$\sigma(N_{\text{true}} + N_{\text{scat}}) = \sqrt{N_{\text{true}} + N_{\text{scat}} + 2N_{\text{random}}}$$

## Corrections; random coincidences; singles method

Random coincidence rate between PMTs  $i$  and  $j$  determined above:

$$\mathcal{R}_{\text{random}}(i, j) = \Delta t \mathcal{R}_{si} \mathcal{R}_{sj}$$

$\mathcal{R}_{\text{random}}(i, j)$  can now be subtracted PMT pair by PMT pair

Singles rate is large, so, correction for random coincidences based on singles has much greater statistical weight than delayed-window method

For the singles method the increase in the statistical (counting) uncertainty is:

$$\sigma(N_{\text{true}} + N_{\text{scat}}) = \sqrt{N_{\text{true}} + N_{\text{scat}} + N_{\text{random}}}$$

To use this method requires that the singles rate is monitored continuously

## Corrections; scatter coincidences

Energy resolution of BGO/LGO used in PET is inferior to that of NaI making the “dual window” approach used in SPECT is inappropriate for PET

Common approaches to the calculation of the scatter correction:

- ① Use the “unscatter-corrected image” and transmission image as input to estimate the rate of scatters  
→ the image is then re-derived from the scatter-corrected image
- ② Use a CT image taken in parallel with the PET image to derive the scatter correction  
→ the image is then re-derived from the scatter-corrected image
- ③ Use events reconstructed outside the object. Such events can only arise due to scatter  
→ the image is then re-derived from the scatter-corrected image

## Corrections; attenuation

Attenuation correction derived for two colinear photons derived for SPECT ... applies here too, so:

$$\mathcal{P}_{\text{coinc}} \propto \exp(-\mu D) \quad (1)$$

where  $\mathcal{P}$  is the probability a coincidence will be formed,  $\mu$  is the attenuation coefficient, and  $D$  is the total thickness of the subject

Equation 1 does not depend on the position of the source between the two PMTs

Attenuation correction can be obtained by taking a “blank” transmission scan with the rod source and a transmission scan with the patient in position. The correction factor for PMTs  $i$  and  $j$ ,  $A_{ij}$  is given by:

$$A_{ij} = \frac{\text{Blnk}_{ij}}{\text{Trns}_{ij}}$$

where  $\text{Blnk}_{ij}$  is the result of the blank scan and  $\text{Trns}_{ij}$  is the result of the patient scan

## Corrections; dead time

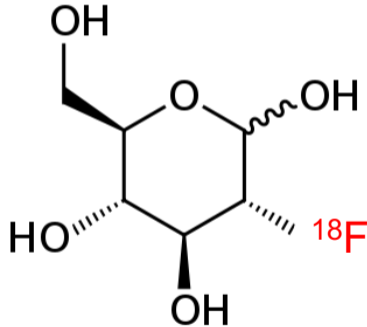
When the scanner is recording a valid coincidence event it is not able to record a second one

This leads to a period during which the scanner is “dead” and is referred to as “dead time”

Dead time is measured empirically using sources of varying activity to determine the fall-off in efficiency as a function of the activity of the source, the size of the object, and the detection threshold

The dead-time correction is then calculated using a computer model based on the measured dead time

# $^{18}\text{F}$ : fluorodeoxyglucose (FDG)

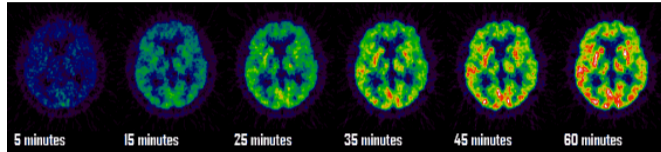


Analogue of glucose

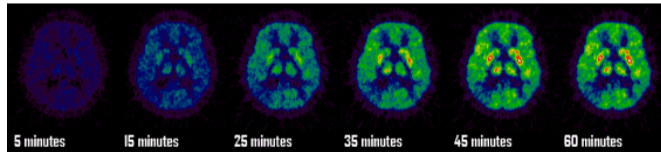
Uptake depends on rate of glucose metabolism

Marker for many disease states and of therapeutic effect

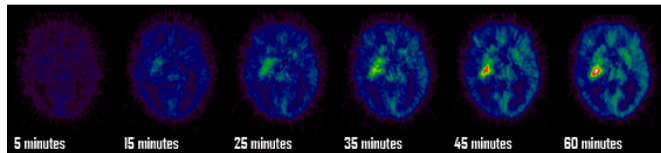
# FDG brain scans



**Healthy patient**  
- normal brain  
metabolism

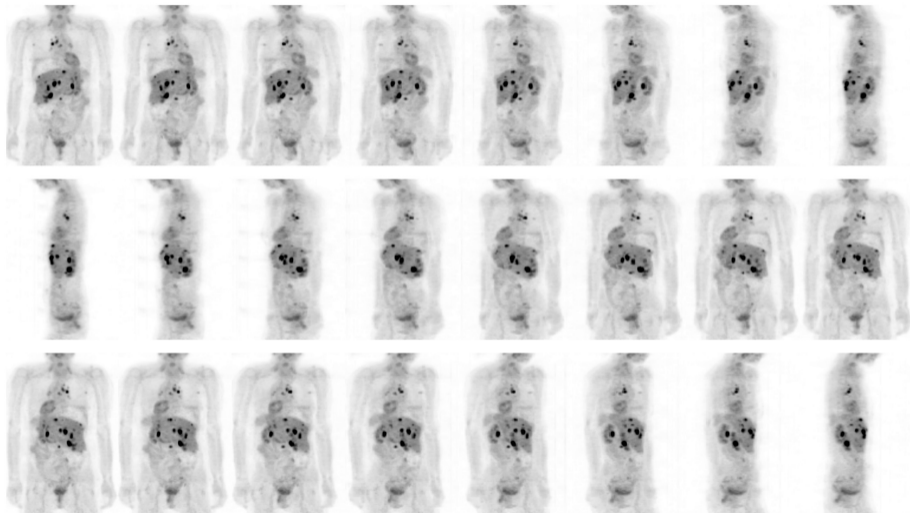


**Alzheimers**



**Brain tumour**

# FDG whole-body imaging



# Cardiac imaging with PET

