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Physics of Medical Imaging and Radiotherapy

Lecture 18; Spatial localisation in MRI

K. Long (k.long@imperial.ac.uk)
Department of Physics, Imperial College London/STFC

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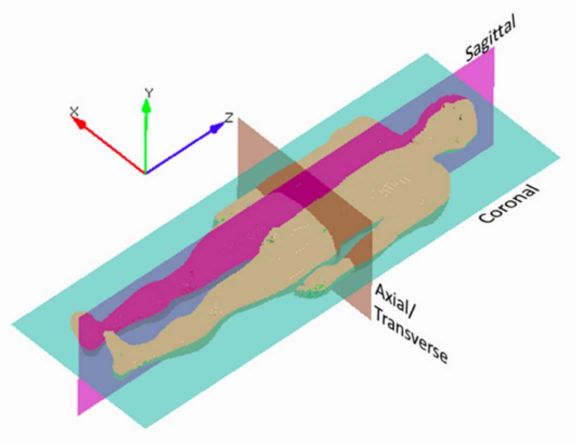
Slice selective excitation

Introduction



Conventional terminology & orientation of RH coordinate system

Contrast between tissues is afforded by RF B_1 pulse sequences such as those discussed above

To make an image, need to localise the signals to appropriately small regions of space

To localise signals exploit:

- Resonance, i.e. Larmor frequency $\nu = \gamma B$
- By making B a function of position

i.e. make ν a function of position:

$$\nu(x, y, z) = \gamma B(x, y, z)$$

Slice selective excitation

Goal: excite a slice of tissue of thickness δ

So far a uniform "main field" $\mathbf{B}_0 = B_0\hat{\mathbf{k}}$ has been considered

Require to make B_z a function of position to make Larmor frequency position dependent

Apply "gradient" fields G_i such that B_z becomes:

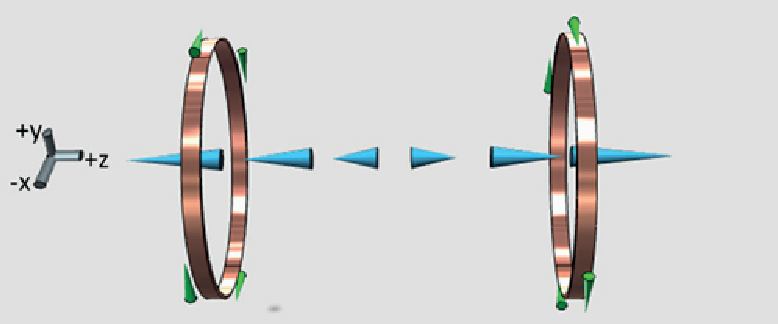
$$B_z(x, y, z, t) = B_0 + xG_x(t) + yG_y(t) + zG_z(t)$$

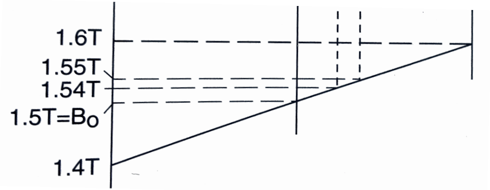
Ideally G_i only have one field component directed along the z direction so that:

$$\mathbf{B} = B_z(x, y, z, t)\hat{\mathbf{k}}$$

With appropriate choice of G_i can generate a field gradient in any direction

Transverse slice; i.e. plane at fixed z





Example:
Helmholtz coils in opposition

Ideal gradient:
G_z = constant

Transverse slice; slice thickness and bandwidth

Lets say that response needs to be isolated to a slice: $\delta z = 5\,\text{mm}$ centred about $z = 0$

Take:

- The magnitude of the main field to be $B_0 = 1.5\,\text{T}$
- The field gradient $G_z = 50\,\text{mT}\,\text{m}^{-1}$
- $\gamma = 42.58\,\text{MHz}\,\text{T}^{-1}$

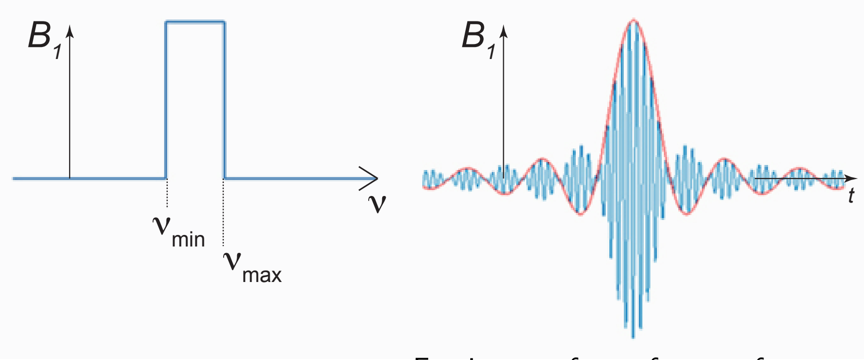
Take the slice to be $-2.5 < z < 2.5\,\text{mm}$, then the Larmor frequency will run over the following range:

$$\begin{aligned}\nu_{\min} &= (1.5 - 0.125 \times 10^{-3}) \times 42.58 \approx 63.8646\,\mathrm{MHz} \\ \nu &= 1.5 \times 42.58 \approx 63.87\,\mathrm{MHz} \\ \nu_{\max} &= (1.5 + 0.125 \times 10^{-3}) \times 42.58 \approx 63.8753\,\mathrm{MHz}\end{aligned}$$

So, the spread of frequencies, the **bandwidth**, $\Delta\nu$ is:

$$\Delta\nu = 63.8646 - 63.8753 \approx 10.7\,\mathrm{kHz}$$

Transverse slice; excitation of spins in slice

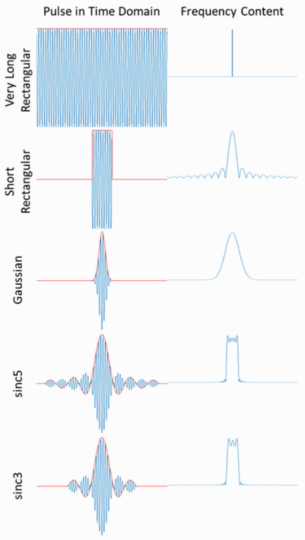


Idealised, square frequency distribution

Fourier transform of square frequency distribution

B_1 oscillates at ν, amplitude is modulated according to “sinc” function (red line)

Transverse slice: excitation pulses



Frequency content of a variety of excitation pulses:

- *Very long rectangular*: narrow band of Larmor frequencies
- *Short rectangular*: frequency distribution follows "sinc" function:

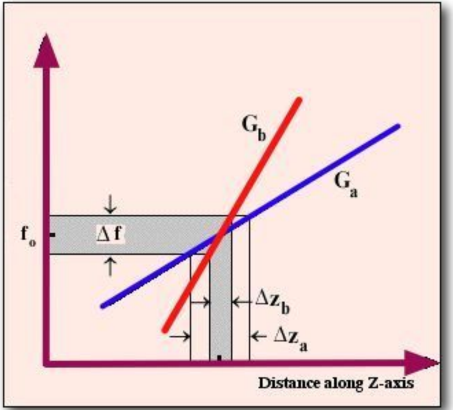
$$A(\nu) \propto \mathrm{sinc}(\nu) = \frac{\sin \nu}{\nu}$$

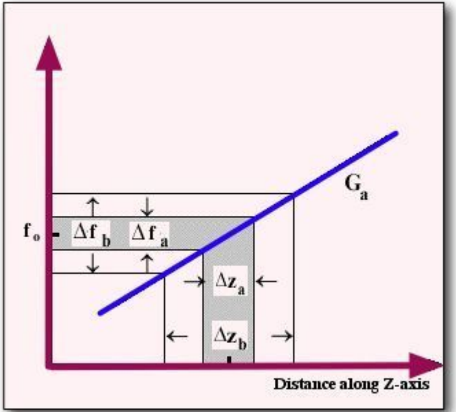
  where $A(\nu)$ is the amplitude of contribution at frequency ν
- *Gaussian*: Fourier transform of Gaussian in t is a Gaussian in ν
- *sincN*: Since square pulse requires contributions over all ν, the frequency range is often truncated. The "sincN" function represents a sinc function for which the frequency range is truncated after N zero crossings

Transverse slice: determining the slice thickness

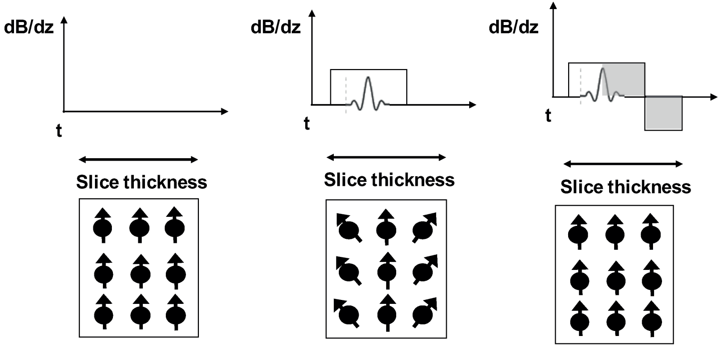
Slice thickness is determined by bandwidth ($\Delta\nu$) and field gradient (G_z)

Sorry for the change in notation!





Transverse slice: spin rephasing pulse



Larmor frequency across slice changes. So, over the time that the gradient pulse is applied, the spins precess at different rates

Therefore, at the end of the pulse the phase of the spins differs as a function of z

A rephasing pulse which reverses the field gradient (i.e. for which $G_z \to -G_z$) is applied

Transverse slice: spin rephasing pulse

Size of the spin rephasing pulse is determined by considering the rate at which the phase difference accumulates

Rate of precession is given by the Larmor frequency, ω, so change in phase of a spin during the gradient pulse is given by:

$$\Phi = \omega\tau = \gamma\left(B_0 + zG_z\right)\tau$$

where τ is the length of the gradient pulse in time

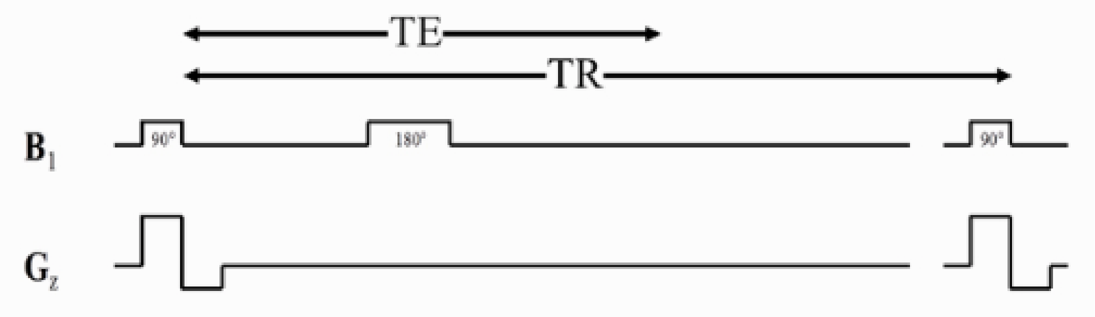
So, phase difference between edges of the slice and the centre is:

$$\Delta\Phi = \gamma\tau G_z \frac{\delta z}{2}$$

So, rephasing pulse, $G_z^{\rm rephase}$, and the length over which it is applied, $\tau^{\rm rephase}$ must satisfy:

$$G_z^{\rm rephase}\tau^{\rm rephase} = G_z\frac{\tau}{2}$$

Transverse slice: partial spin-echo pulse sequence



B_1 rotates net magnetisation in the selected slice with gradient pulse applied

Summary of section 3

Localisation of MRI signal to plane achieved using magnetic-field gradient combined with frequency-dependent readout of radiowave generated by precession of net magnetisation

Frequency content of RF (B_1) pulse determines spread of perturbations to Larmor frequency

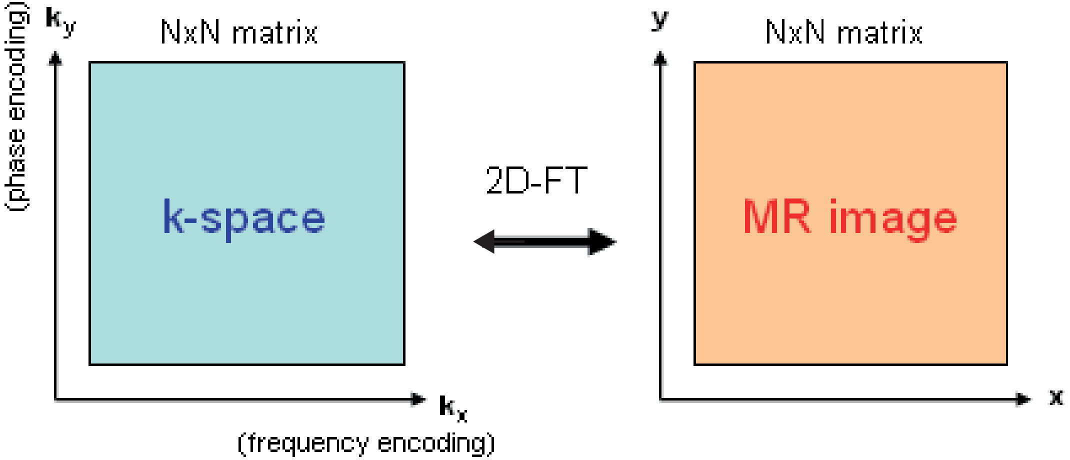
Spin-rephasing pulse applied after main B_1 pulse. Length of rephasing pulse required is half that of the main B_1 pulse

Section 2

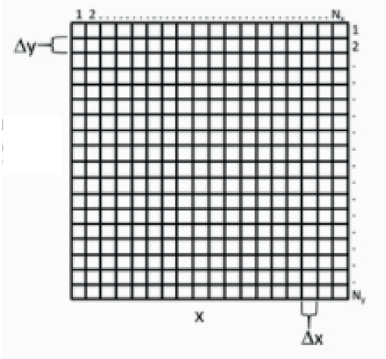
Encoding spatial information in k-space

Encoding spatial information into the net magnetisation

The basis is a 2D Fourier transform:



2D Fourier transformation



2D image in "coordinate space", x, y, presented in pixel grid

Field of view, FOV, in coordinate space:

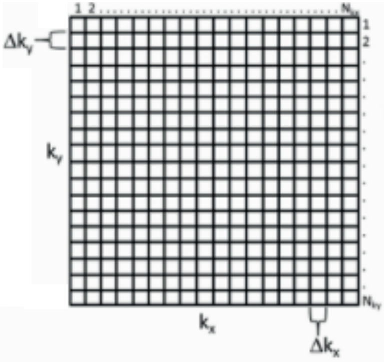
$$(x_{\text{max}} - x_{\text{min}}, y_{\text{max}} - y_{\text{min}})$$

Pixel size (resolution):

$$\Delta x = \frac{x_{\text{max}} - x_{\text{min}}}{N_x}$$

$$\Delta y = \frac{y_{\text{max}} - y_{\text{min}}}{N_y}$$

2D Fourier transformation



2D image in "k space", k_x, k_y, presented in pixel grid

Field of view, FOV, in k space:

$$(k_{x\,\max} - k_{x\,\min}, k_{y\,\max} - k_{y\,\min})$$

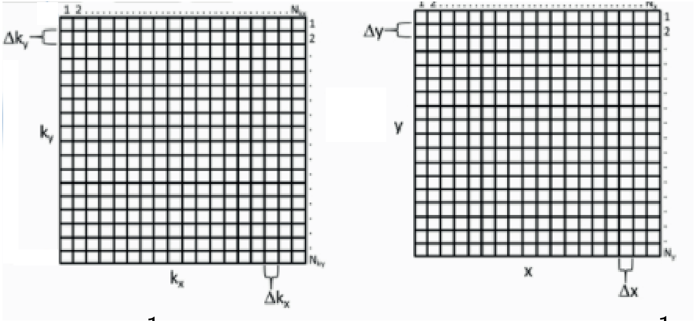
Pixel size (resolution):

$$\Delta k_x = \frac{k_{x\,\max} - k_{x\,\min}}{N_x}$$

$$\Delta k_y = \frac{k_{y\,\max} - k_{y\,\min}}{N_y}$$

2D Fourier transformation

Transformation between resolution in coordinate-space and k-space representations:



$$\Delta k_x = \frac{1}{(x_{\max} - x_{\min})} \qquad \Delta x = \frac{1}{(k_{x\,\max} - k_{x\,\min})}$$

$$\Delta k_y = \frac{1}{(y_{\max} - y_{\min})} \qquad \Delta y = \frac{1}{(k_{y\,\max} - k_{y\,\min})}$$

2D Fourier transformation

Define $\rho(x, y)$ to be the intensity pixel-by-pixel in coordinate space.

2D Fourier transform from coordinate to k space is then:

$$S\left(k_x, k_y\right) = \int_{y_{\min}}^{y_{\max}} \int_{x_{\min}}^{x_{\max}} \rho\left(x, y\right) \exp\left(-i2\pi k_x x\right) \exp\left(-i2\pi k_y y\right) dxdy$$

where $S(k_x, k_y)$ is the intensity pixel-by-pixel in k space

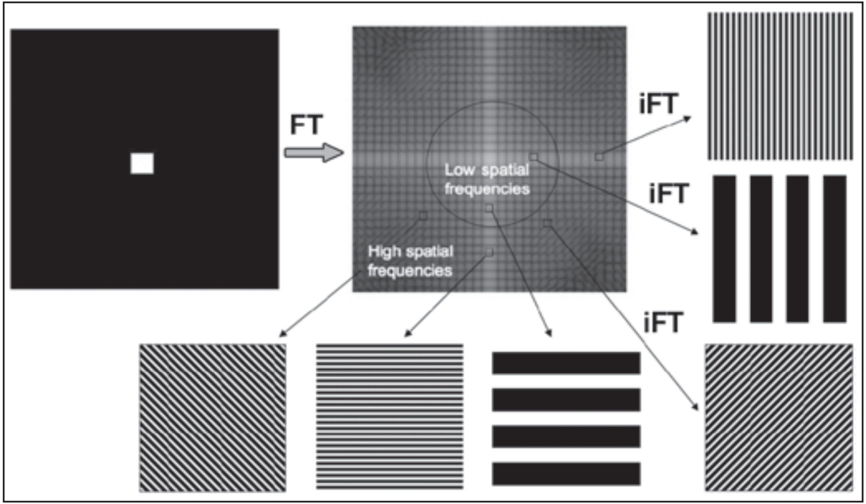
Inverse Fourier transform takes k-space intensity map to coordinate-space intensity map:

$$\rho\left(x, y\right) = \int_{k_{y\,\min}}^{k_{y\,\max}} \int_{k_{x\,\min}}^{k_{x\,\max}} S\left(k_x, k_y\right) \exp\left(i2\pi k_x x\right) \exp\left(i2\pi k_y y\right) dk_x dk_y$$

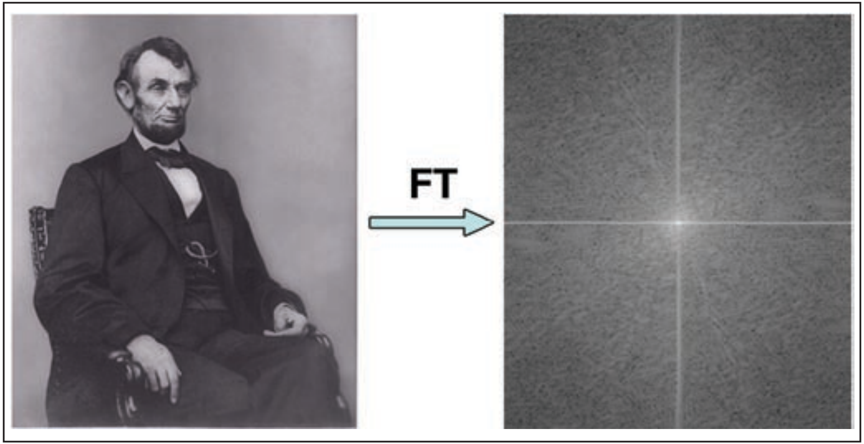
Example one: a single dot

Example two: three dots

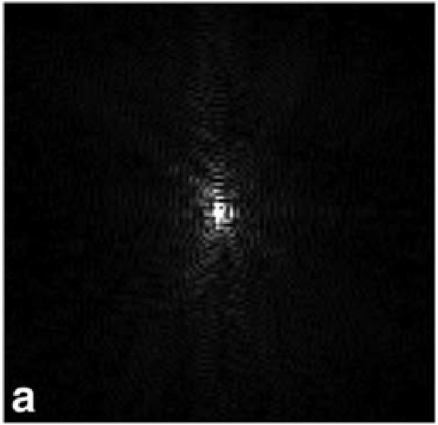
Example three: Square in centre of field of view



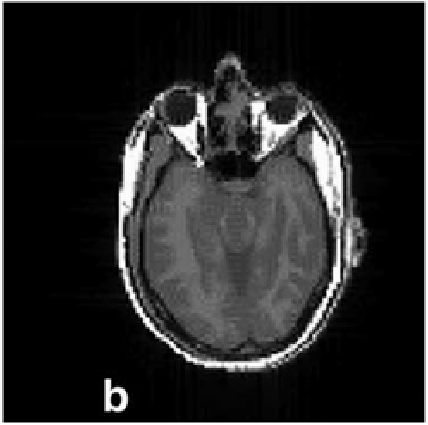
Example four: Abraham Lincoln



Example five: Section through skull



(a) *k*-space image of head



(b) coordinate-space image of head

Challenge: record *k*-space image using NMR signals

Summary of section 4

Intensity distribution in "coordinate space" ($\rho(x, y)$) mapped using a Fourier transform onto intensity distribution in "k"-space ($S(k_x, k_y)$)

Signals generated in MRI scan recorded in k-space; coordinate space image obtained by inverse Fourier transform

Section 3

Encoding spatial information into net magnetisation

Spatial encoding and field gradients

Gradient pulse causes Larmor frequency to become a function of position:

$\rightarrow$ z-gradient pulse used with modulated B_1 pulse for slice selection

The *frequency and phase* of the nuclear precession will become a function of position over the period of a gradient pulse

Exploit these features to:

- Encode x position into k_x via "frequency encoding"
- Encode y position into k_y via "phase encoding"

Remember, gradient pulses G_i are such that:

$$B_z(x, y, z, t) = B_0 + xG_x(t) + yG_y(t) + zG_z(t)$$

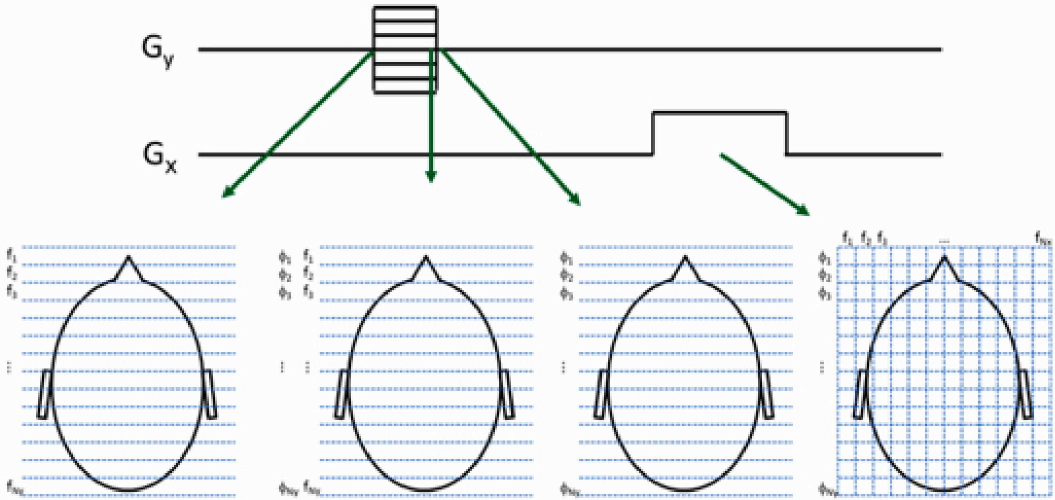
$G_x = \frac{\partial B_z}{\partial x}$; i.e. a magnetic-field gradient in x direction

magnetic field xG_x is in the $\hat{\mathbf{k}}$ direction

$G_y = \frac{\partial B_z}{\partial y}$; i.e. a magnetic-field gradient in y direction

magnetic field yG_y is in the $\hat{\mathbf{k}}$ direction

Conversion of field gradient into *k* space

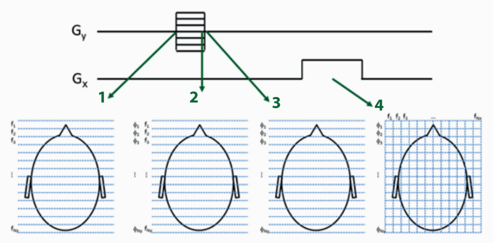


Conversion of field gradient into k space

Example:

phase encode y,

frequency encode x



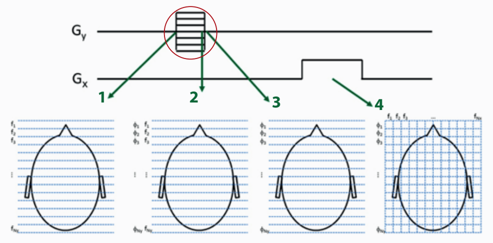
1. At start of phase encoding-pulse, spins are in phase. G_y causes Larmor frequency to be function of y: $\nu = f(y)$
2. At end of phase encoding pulse, phase of precession, ϕ, has become a function of y, i.e. $\phi \to \phi(y)$
3. As time passes, phase dependence on y is preserved, i.e. $\phi = \phi(y)$
4. Gradient pulse G_x causes Larmor frequency to become a function of x. Result is that y-position information is encoded in $\phi = \phi(y)$ and x-position information is encoded in $\nu = f(x)$

Spatial encoding gradient pulses part of pulse sequence

Example:

phase encode y,

frequency encode x



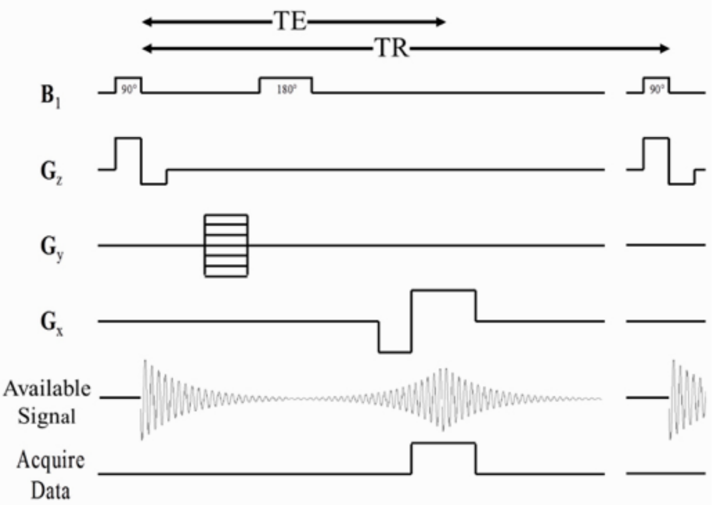
Phase- and frequency-encoding pulses part of a longer pulse sequence that repeats with period TR

At each repeat the amplitude of G_y, the phase-encoding pulse, has a different amplitude (as indicated on the figure)

For example:

- 1^{st} iteration of sequence: $G_y = 0$;
- 2^{nd} iteration of sequence: $G_y = +\eta$;
- 3^{rd} iteration of sequence: $G_y = -\eta$;
- ...

Example pulse sequence



Example of spin-echo pulse sequence

Data is acquired at spin-echo time as shown

Combination of phase and frequency encoding pulses and repetition to obtain N_y data points completes ones transverse slice

Summary of section 1

Field gradient makes Larmor *frequency* a function of position;

Phase difference as a function of position develops during application of gradient pulse

Exploit the position dependence of frequency and phase to encode image in k-space

Pulse sequences designed to optimise contrast within slice for various tissues and types of investigation