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Physics of Medical Imaging and Radiotherapy

Lecture 2; Generation of contrast

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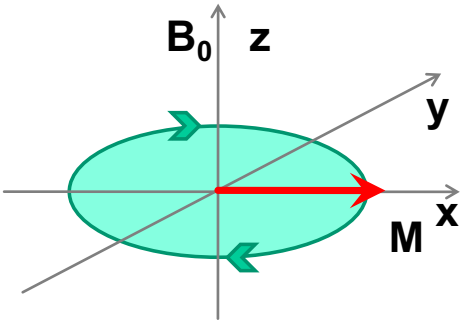
Section 1

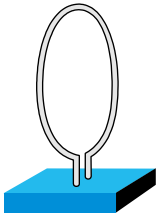
Free induction decay

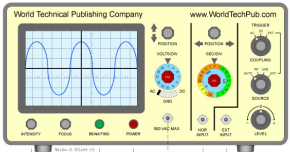
Detection of signal precession of magnetisation vector

M rotated using B_1 RF pulse. If flip angle α is not a multiple of 180°, then, result of B_1 pulse is a component of magnetisation in the x, y plane that is precessing

This yields an RF wave that can be detected

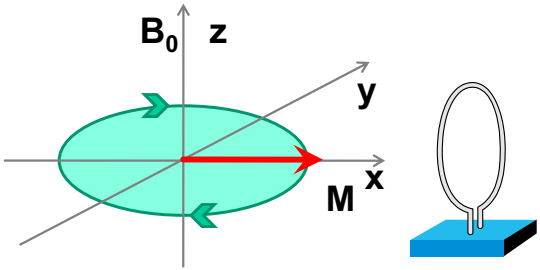


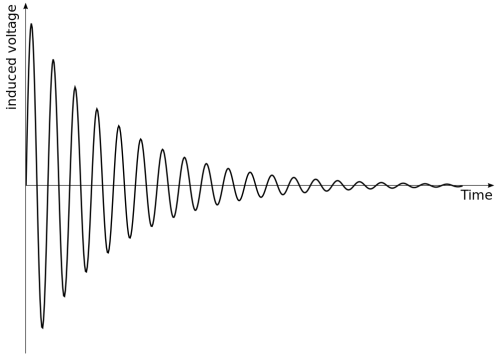




Free induction decay (FID)

Occurs when perturbing field (B_1) is turned off





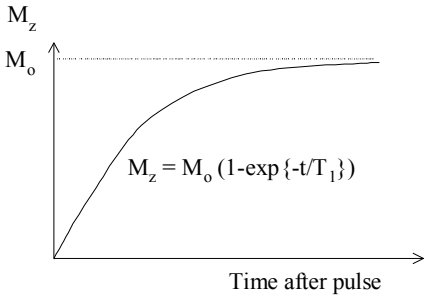
Note; exponential decay of amplitude of transverse magnetisation. Frequency of rotation remains the Larmor frequency corresponding to B_0

Spin-lattice (longitudinal) relaxation

When the B_1 pulse is turned off, the longitudinal magnetisation, M_z, recovers:

$$\frac{dM_z}{dt} = \frac{M_0 - M_z}{T_1} \qquad \Rightarrow M_z(t) = M_0\left[1 - \exp\left(-\frac{t}{T_1}\right)\right]$$

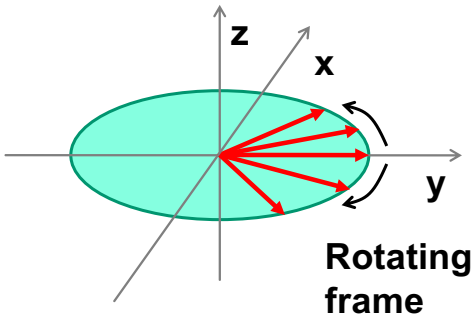
The process is characterised by a time constant T_1



Spin-lattice relaxation:

- ^{1}H spins relax to the low-energy state. Energy released returns to the "lattice" as heat
- Relatively ineffective thermal coupling to ^{1}H nuclei results in T_1 being large, typically $T_1 > 200\,\text{ms}$

Spin-spin (transverse) relaxation



Contributions to M_{xy} smear out (decohere) rapidly

Causes M_{xy} to decay quickly

Some factors that affect the decoherence rate:

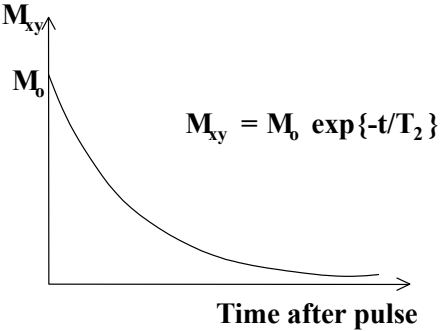
- Resonance frequency changes due to local magnetic fields
- Thermal excitations
- Spin "mobility"
- Presence of large molecules, paramagnetic ions or molecules, outside interference

Spin-spin (transverse) relaxation

When the B_1 pulse is turned off, transverse magnetisation, decays:

$$\frac{dM_{xy}}{dt} = -\frac{M_{xy}}{T_2} \qquad \Rightarrow M_{xy}(t) = M_0 \exp\left(-\frac{t}{T_2}\right)$$

The process is characterised by a time constant T_2



Spin-spin relaxation:

- ^{1}H spins interact magnetically with their neighbours
- Coupling causes a variety of magnetic fields, causing a variety of precessions
- Effective randomisation of precessional modes leads to efficient depolarisation in transverse plane
- Results in T_2 being comparatively small, typically $T_2 \lesssim 100\,\text{ms}$

Relaxation times for a variety of tissues

Tissue Type	T1 (ms)	T2 (ms)
Adipose tissues	240-250	60-80
Whole blood (deoxygenated)	1350	50
Whole blood (oxygenated)	1350	200
Cerebrospinal fluid (similar to pure water)	4200 - 4500	2100-2300
Gray matter of cerebrum	920	100
White matter of cerebrum	780	90
Liver	490	40
Kidneys	650	60-75
Muscles	860-900	50

Relaxation times characteristic of tissue type

For materials important for human imaging $T_1 > T_2$

Bloch equation revisited

Block equation may now be updated to include FID:

$$\frac{d\mathbf{M}}{dt} = \gamma\left(\mathbf{M} \times \mathbf{B}_0\right) - \frac{\mathbf{M}_{xy}}{T_2} + \frac{M_0 - M_z}{T_1}\hat{\mathbf{k}}$$

where:

- The first term describes the torque produced by the main (solenoid) field $\mathbf{B}_0$
- The second term describes the evolution of the transverse magnetisation vector $\mathbf{M}_{xy}$ due to the spin-spin interaction; time constant T_2
- The third term describes the evolution of the longitudinal magnetisation M_z due to the spin-lattice interaction; time constant T_1
- M_0 is the net magnetisation at equilibrium aligned with and proportional to $\mathbf{B}_0$

Complication: additional factors affecting the decay of the transverse magnetisation

T_2, the intrinsic spin-spin relaxation time is determined by non-reversible thermodynamic processes at the nuclear level.

The spin-spin relaxation time constant is reduced by a number of factors. A significant contribution comes from inhomogeneities in the main field $\mathbf{B}_0$

Inhomogeneities give rise to reversible thermodynamic processes. The associated relaxation of the transverse magnetisation is characterised by a time constant T_2'

The effective spin-spin relaxation time constant, T_2^*, is given by:

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

$T_2' < T_2$ and so $T_2^* < T_2$. Need to develop techniques to recover T_2 as this carries the clinically-relevant information

Comparison of T_2 and T_2'

T_2

- The individual dipoles that sum up to produce the transverse magnetization are not precessing at precisely the same rate
- As a water molecule tumbles due to thermal motions, each H nucleus feels a small, randomly varying magnetic field in addition to B_0
- When the random field adds to B_0, the dipole precesses a little faster, and when it subtracts from B_0, it precesses a little slower
- For each nucleus the pattern of random fields is different, so as time goes on the dipoles get progressively more out of phase with one another, and as a result no longer add coherently

T_2'

- The source of this T_2' effect is magnetic field inhomogeneity
- Because the precession frequency of the local transverse magnetization is proportional to the local magnetic field, any field inhomogeneity will lead to a range of precession rates
- Over time the precessing magnetization vectors will get out of phase with one another so that they no longer add coherently to form the net magnetization
- As a result, the net signal is reduced because of this destructive interference
- Static field offsets rather than fluctuating fields

Summary of section 1

Rotated net-magnetisation vector relaxes back to equilibrium orientation with time constant, T_1; spin-lattice relaxation time constant

Projection of net magnetisation vector in x, y plane decays away with time constant, T_2; spin-spin relaxation time constant

The effective spin-spin time constant, T_2^*, is a combination of the intrinsic spin-spin relaxation time constant (T_2) and the effect of "instrumental" effects such as inhomogeneities in the applied magnetic field (T_2')

Section 2

Determination of the spin-lattice relaxation time, T_1

Relaxation times revisited

Tissue Type	T1 (ms)	T2 (ms)
Adipose tissues	240-250	60-80
Whole blood (deoxygenated)	1350	50
Whole blood (oxygenated)	1350	200
Cerebrospinal fluid (similar to pure water)	4200 - 4500	2100-2300
Gray matter of cerebrum	920	100
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Muscles	860-900	50

Relaxation times characteristic of tissue type

For materials important for human imaging $T_1 > T_2$

T_1 characteristic of recovery of longitudinal magnetisation

T_2 must be extracted from the decay of the transverse magnetisation which is characterised by T_2^* which is related to T_2 by:

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2'}$$

What does it take to make an MRI image

NMR can be used to generate signals that depend on the concentration of ^{1}H in tissue; the basis of an imaging technique

The spin-lattice and spin-spin relaxation times, T_1 and T_2 respectively, depend on tissue type—so can be used to distinguish neighbouring tissues

To generate an image need to:

- Extract T_1 and T_2 – the basis of the generation of contrast; and
- Spatially localise the signal

Free induction decay revisitted

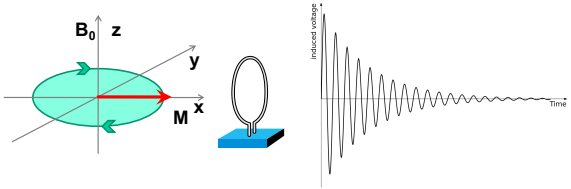
System set up in equilibrium; net magnetisation, $\mathbf{M}_0$, parallel to $\mathbf{B}_0$ and of magnitude M_0

90° RF magnetic field pulse applied to rotate net magnetisation, $\mathbf{M}_0$, into x, y plane

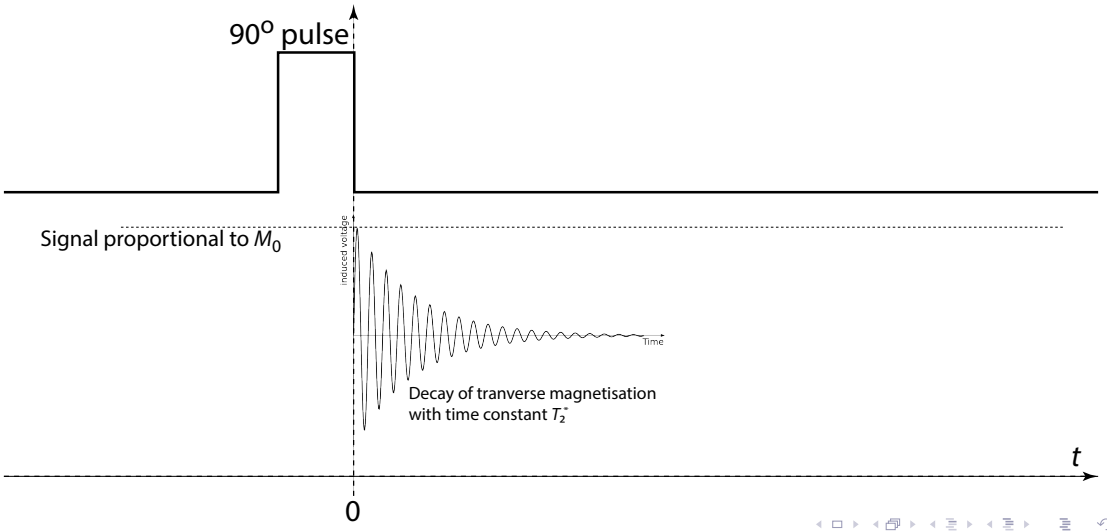
Take $t = 0$ to be time at which 90° pulse ends. Magnitude of transverse magnetisation, M_{xy}, at $t = 0$:

$$M_{xy}(t = 0) = M_{xy}(0) = M_0$$

M_{xy} decays exponentially, as described in lecture 1



Application of 90° pulse and spin-spin relaxation time, T_2



Recovery of longitudinal polarisation and spin-lattice relaxation time, T_1

Longitudinal magnetisation recovers according to:

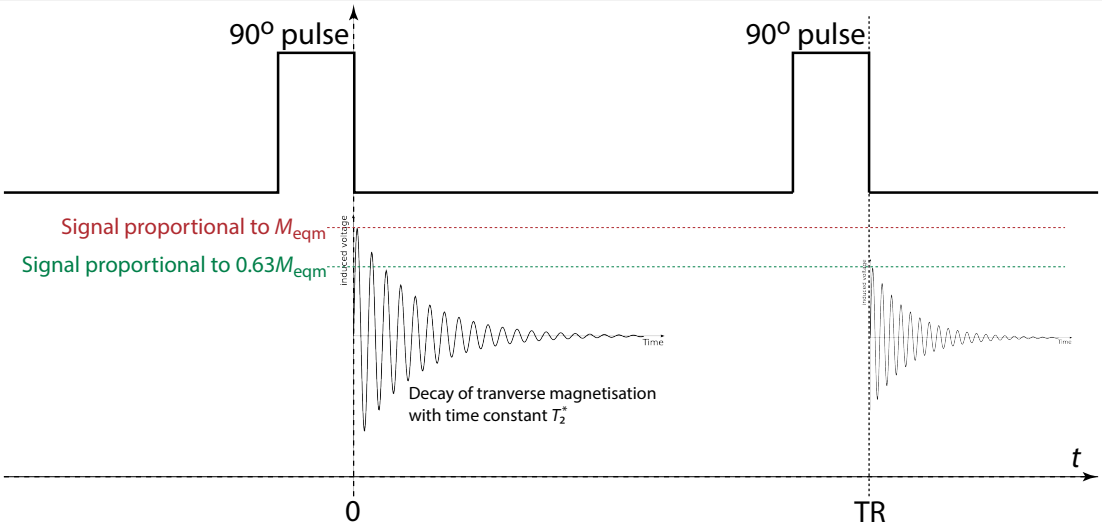
$$M_z(t) = M_0 \left[1 - \exp\left(-\frac{t}{T_1}\right)\right]$$

So, for $t \gtrsim 5T_1$, $M_z - M_0 \lesssim 0.5\%$, i.e. longitudinal magnetisation has recovered

If a second 90° pulse is applied for $t < 5T_1$ then the resulting M_{xy} will be less than M_0

For example, if the second 90° pulse is applied at $t = T_1$, then $M_{xy}(t = T_1) = 0.63M_0$

Applicaton of multiple 90° pulses



The time to repetition, TR, and extraction of T_1

Longitudinal magnetisation recovers according to:

$$M_z(t) = M_0 \left[1 - \exp\left(-\frac{t}{T_1}\right)\right]$$

So, for $t \gtrsim 5T_1$, $M_z - M_0 \lesssim 0.3\%$, i.e. longitudinal magnetisation has recovered

If a second 90° pulse is applied for $t < 5T_1$ then the resulting M_{xy} will be less than M_0

In general:

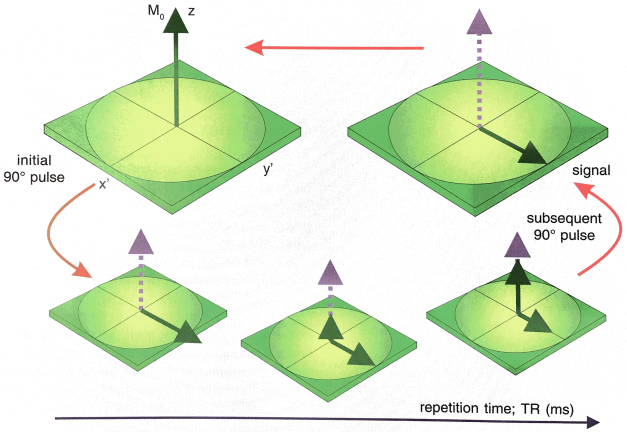
$$M_z(\mathrm{TR}) = M_0 \left[1 - \exp\left(-\frac{\mathrm{TR}}{T_1}\right)\right]$$

So, repetition of 90° pulse at $t = \mathrm{TR}$ gives $M_{xy}(\mathrm{TR}) = M_z(\mathrm{TR})$

Can extract T_1 by measuring $M_{xy}(\mathrm{TR})$ as a function of TR

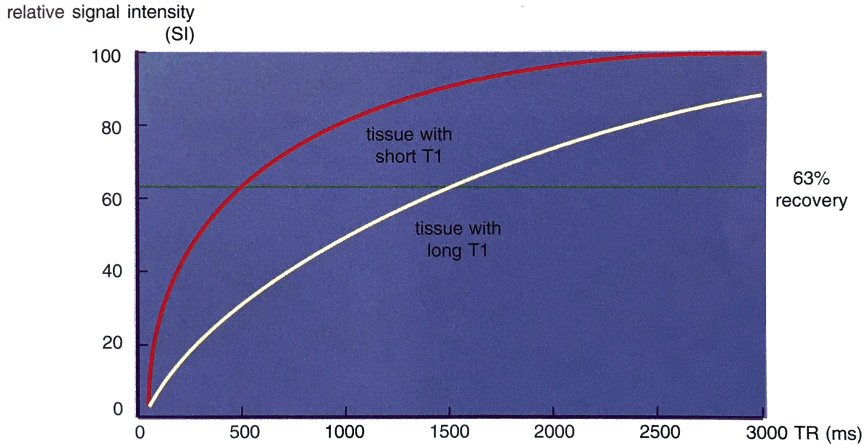
Partial saturation pulse sequence

"**Partial saturation pulse sequence**", graphical representation of evolution of magnetisation

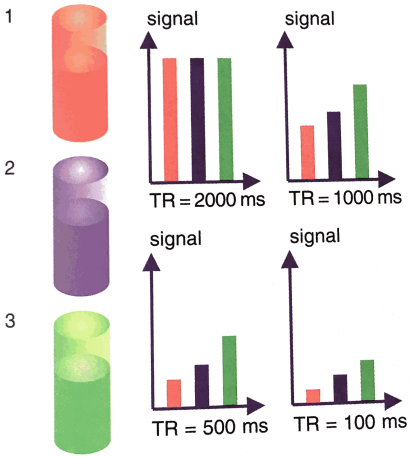


Extraction of spin-lattice relaxation time constant, T_1

Comparison of relative signal intensity in partial saturation sequence for two different tissues



Using TR to distinguish between different tissues



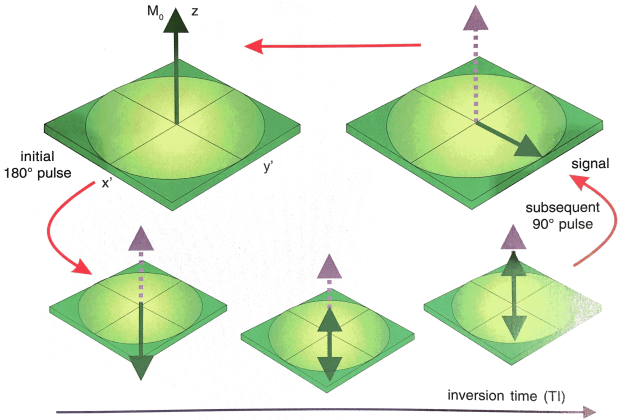
Example three types of tissue:

1. Blood: $T_1 = 1350\,\text{ms}$
2. Muscle: $T_1 = 875\,\text{ms}$
3. Fat: $T_1 = 230\,\text{ms}$

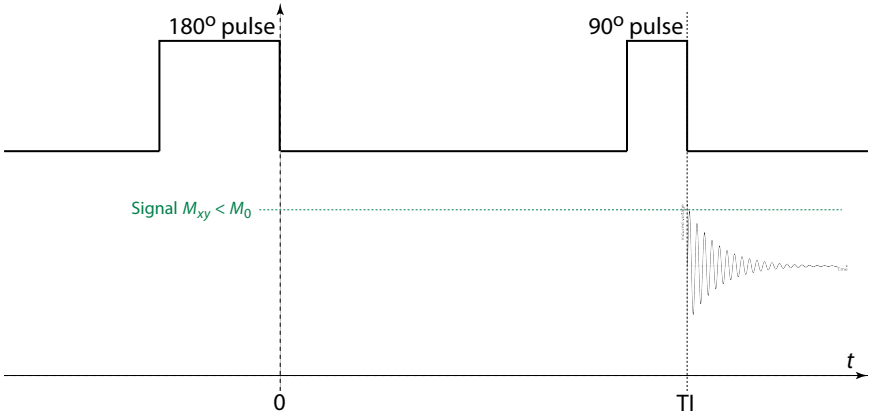
Note how tissues can be distinguished by comparing signal behaviour as a function of TR

Inversion recovery pulse sequence and time to inversion, TI

"**Inversion recovery pulse sequence**", graphical representation of evolution of magnetisation



Inversion recovery: using TI to extract T_1



$$M_z(\mathrm{TI}) = M_0 \left[1 - 2\exp\left(-\frac{\mathrm{TI}}{T_1}\right)\right]$$

Summary of section 2

T_1, the longitudinal or spin-lattice, relaxation time constant can be reconstructed using pulse sequences in which the net magnetisation is repeatedly rotated into the x, y plane and the evolution of the maximum of the transverse magnetisation is observed

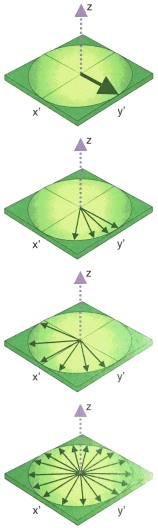
Pulse sequences used to obtain T_1:

- 90° pulse sequence
- Inversion recovery pulse sequence

Section 3

Determination of the spin-spin relaxation time, T_2

Spin-spin relaxation time, T_2



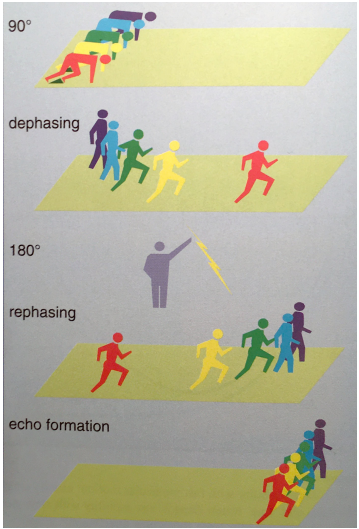
System set up in equilibrium; net magnetisation, $\mathbf{M}_0$, parallel to $\mathbf{B}_0$ and of magnitude M_0

90° RF magnetic field pulse applied to rotate net magnetisation, $\mathbf{M}_0$, into x, y plane

Take $t = 0$ to be time at which 90° degree pulse ends. The net magnetisation is precessing around $\mathbf{B}_0$

Rate of precession of individual ^{1}H nuclei depends on local magnetic environment: some precess faster, some slower. Results in decoherence, time constant T_2^*

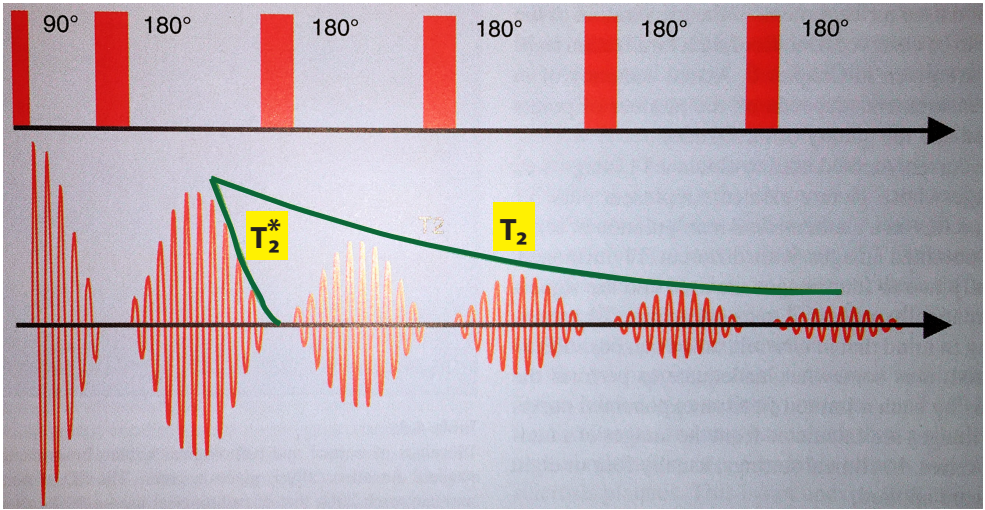
Spin-spin relaxation time, T_2



Before "doing the spins", an analogy:

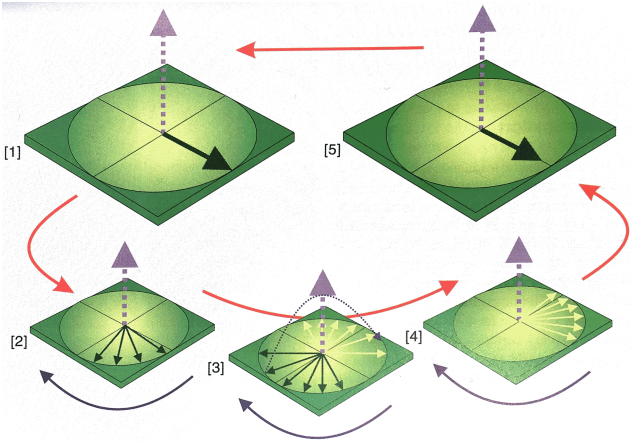
- A set of sprinters have been prepared at the starting line
- The "starting gun" is the end of the 90° pulse
- The sprinters run for a period of time, t_{run}
- At t_{run} the sprinters' phase is rotated by 180°:
  The first becomes the last, etc.
- After a further t_{run} all sprinters are back in line
- The line of sprinters at $t = 2t_{\mathrm{run}}$ is an "echo" of the situation at $t = 0$

Spin-spin relaxation time, T_2



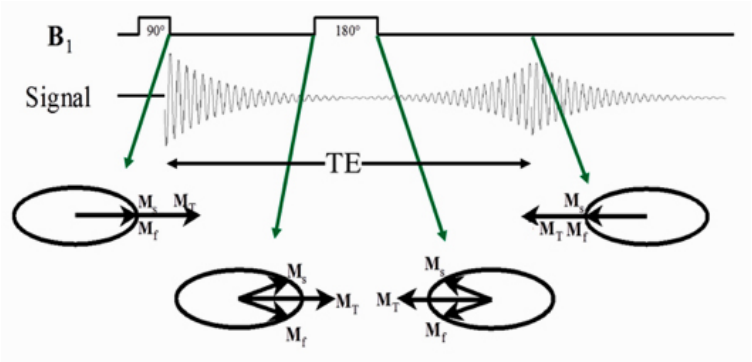
The spin-spin relaxation time constant, T_2

"**Spin echo sequence**", graphical representation of evolution of magnetisation

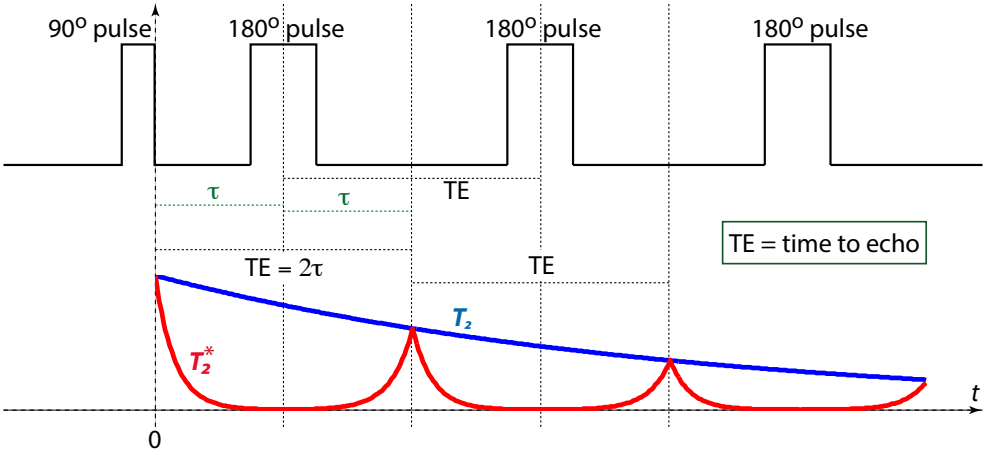


The spin-spin relaxation time constant, T_2

"**Spin echo sequence**", graphical representation of evolution of magnetisation



Spin-spin relaxation time, T_2



$$M_{xy}(\mathrm{TE}) = M_0 \exp\left(-\frac{\mathrm{TE}}{T_2}\right)$$

Summary of section 3

T_2, the spin-spin, relaxation time constant can be reconstructed using a spin-echo pulse sequence