Centre for the Clinical Application of Particles

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Physics of Medical Imaging and Radiotherapy

Lecture 1; Introduction to MRI and quantum-mechanical foundations

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Contents

Section 1

Introduction to MRI

‘Guilt-free’ imaging

Whole-body imager, Star Trek style

Nuclear diagnostics and X-ray imaging:

- Image constructed using ionising radiation
- Necessarily delivers dose to patient
- Dose implies risk of initiating disease

Magnetic resonance imaging (MRI):

- Image generated by exploiting magnetic moment of H nuclei
- Patient immersed in magnetic field
- No permanent harmful effects reported

Nuclear magnetic moment

Proton (and neutron) magnetic moment:

- Nucleons each have spin $\frac{1}{2}$
- Magnetic moment generated by nuclear charge
  - Contributions to nuclear spin arise from quarks and gluons
  - Quantitative explanation of nuclear magnetic moment is active field of research
- Magnetic moment, $\boldsymbol{\mu}$, is related to nuclear spin, $\mathbf{s}$ by:

$$\boldsymbol{\mu} = \gamma \mathbf{s}$$

where γ is the “gyromagnetic” ratio

Nuclear magnetic resonance

Effect of uniform magnetic field **B**:

- **B** provides "quantisation axis":
  $\Rightarrow$ nuclear dipoles align with magnetic field
- Proton spin is $\frac{1}{2}$, so only two states:
  Spin "up" and spin "down"
- Energy splitting; 2 energy levels:
  - Lower energy level has magnetic moment parallel to magnetic field
  - Higher energy level has magnetic moment anti-parallel to magnetic field
- Resonance:
  - Call energy splitting ΔE
  - Transitions between the two energy levels cause absorption or emission of electromagnetic (em) radiation for which $\Delta E = h\nu$
  - Resonance occurs when em radiation of frequency ν is injected

Magnetic resonance imaging

Magnetic resonance imaging (MRI) exploits this resonance

Steps:

- Apply uniform magnetic field, align proton (^{1}H) spins
- Apply radiation, at exactly ν, cause transitions between "spin up" & "spin down" states
- Turn off the radiation . . . and . . .
- "Listen" for radiation at exactly ν as the spins realign

Brilliant!
Simple principle and elegant technique, exploited in exquisitely sophisticated imaging systems.

The physical principles

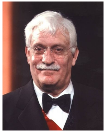
1938: I. Rabi: Discovered nuclear magnetic resonance
Nobel Prize 1944

1946: F. Bloch & E. Purcell: Developed methods that allow precision methods using NMR
Nobel Prize 1952

1955/56: E. Odeblad & G. Lindström: Applied NMR to living cells from animal tissue

1968: J.A. Jackson and W.H. Langham: First NMR measurements from living animals

Cancerous and normal cells differ



Raymond Damadian

Relaxation times that characterise recovery of ground-state magnetisation shown to differ between normal and tumour cells

Tumor Detection by Nuclear Magnetic Resonance
Author(s): Raymond Damadian
Source: *Science*, New Series, Vol. 171, No. 3976 (Mar. 19, 1971), pp. 1151-1153
Published by: American Association for the Advancement of Science
Stable URL: https://www.jstor.org/stable/1730608
Accessed: 01-03-2020 09:22 UTC

REFERENCES
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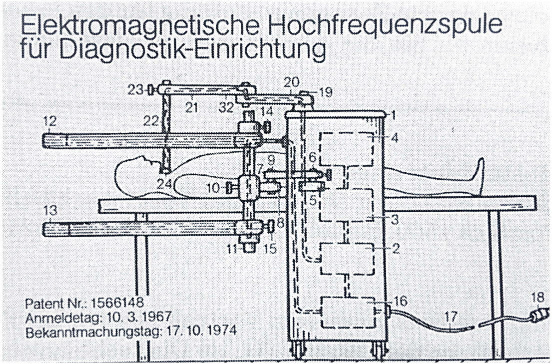
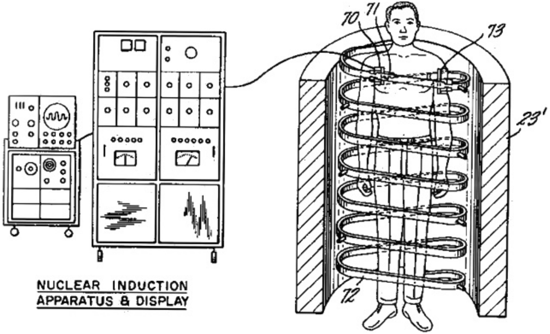
You may need to log in to JSTOR to access the linked references.

Early proposals for MRI scanners

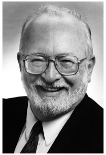
Alexander Ganssen; patent 1967



Raymond Damadian; patent 1972



Spatial localisation using magnetic-field gradients



Paul Lauterbur

Superimpose field gradient on main uniform magnetic field. Incident em radiation at frequency ν only resident in a particular location in subject

Nature Vol. 242 16 March 1973

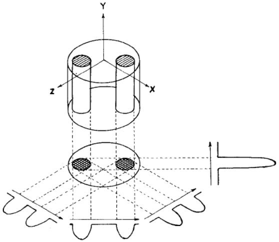


Fig. 1 Relationship between a three-dimensional object, its two-dimensional projection along the Y-axis, and four one-dimensional projections at 45° intervals in the XZ-plane. The arrows indicate the gradient directions.

Fig. 2 Proton nuclear magnetic resonance zeugmatogram of the object described in the text, using four relative orientations of object and gradients as diagrammed in Fig. 1.

Rapid, “snap-shot” MRI

Peter Mansfield

Use of “echo planar imaging” to allow fast “snap-shot” imaging required active screening of fields created by currents induced in cryostat walls

P. Mansfield, Nobel Lecture 2003

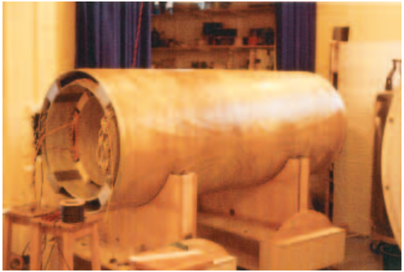


Figure 2. Photograph of a doubly screened active magnetic shielded gradient coil set for insertion in the super-conductive magnet of Figure 1.

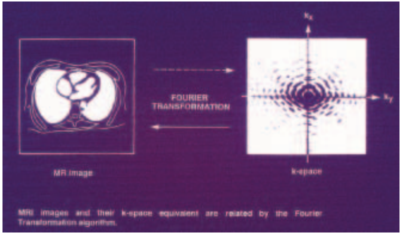


Figure 3. Diagram of a slice through the mediastinum showing the two lung fields and heart mass, also shown is the Fourier transform of this real-space image to the k-space map. (Reproduced with permission from M K Stehling, R Turner and P Mansfield, SCIENCE 253, 43–50 (1991).)

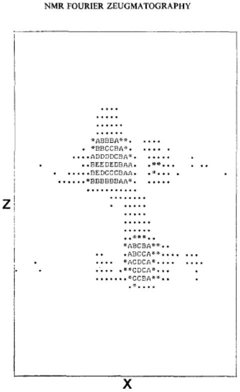
NMR zeugmatography

1975: A. Kumar, D. Welti, R. Ernst

Application of Fourier techniques to the reconstruction of images

Journal of Magnetic Resonance, Vol 18, P 69–83(1975)

zeug·ma·tog·ra·phy (zūg'mă-tog'ră-fē),
Term coined by Lauterbur in 1972 for the joining of a magnetic field and spatially defined radiofrequency field gradients to generate a two-dimensional display of proton density and relaxation times in tissues, the first nuclear magnetic resonance image.



State of the art

Summary of section 1

Magnetic moment of proton exploited to provide energy splitting, ΔE, between spin-up and spin-down states in applied magnetic field

Injection of radio-frequency wave with a frequency that resonates with the splitting then used to manipulate population of protons in the spin up and spin down states

Images produced by manipulating applied magnetic field and frequency of RF field gradients

Section 2

Quantum mechanical foundations

Theoretical description; a hybrid of quantum and classical

Nuclear magnetic resonance & MRI are both inherently quantum mechanical effects:

- Signal is generated by manipulating the *spins* of hydrogen nuclei:
  - Spin is postulated to explain hyperfine structure, Stern-Gerlach experiment, ...
  - Understood theoretically through the symmetries of space and time
- Magnetic moment of proton, $\boldsymbol{\mu}$, is related to the proton spin, $\mathbf{s}$, by:

$$\boldsymbol{\mu} = \gamma \mathbf{s}$$

  where γ is the "gyromagnetic ratio"

Hybrid, quantum/classical treatment:

- Quantum mechanics: energy splitting and population in ground and excited state
- Classical: magnetisation vector, its precession, and the manipulation of the magnetisation vector to understand the signals used for imaging

Interaction of nuclear magnetic dipole with uniform magnetic field

The contribution, $\delta\mathcal{U}$, to the potential energy of a proton immersed in a magnetic field, $\mathbf{B}_0$, is given by:

$$\delta\mathcal{U} = -\mathbf{B}_0 \cdot \boldsymbol{\mu}$$

Lets consider a proton which, in the absence of a magnetic field, has energy E. Applying the magnetic field introduces $\delta\mathcal{U}$ into the Schrödinger equation resulting in a splitting of the proton energy level such that $E \rightarrow E'$ given by:

$$E' = E \pm E_{m_s}$$

where

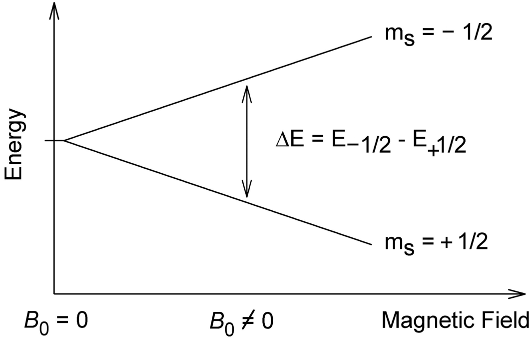
$$E_{m_s} = -m_s \gamma \hbar B_0$$

where m_s is the quantum number associated with the component of the proton spin parallel to $\mathbf{B}_0$, $\hbar$ is Planck's constant divided by 2π, and B_0 is the magnitude of $\mathbf{B}_0$

For the proton:

$$m_s = \pm\frac{1}{2}$$

Larmor equation



ΔE, splitting between two levels with $m_s = \pm\frac{1}{2}$:

$$\Delta E = \gamma \hbar B_0$$

Planck's law relates energy splitting to the angular frequency, ω_0, of the radiation required to excite the transition, therefore:

$$\Delta E = \hbar \omega_0$$

Writing ω_0 in terms of γ and B_0 yields the Larmor equation:

$$\omega_0 = \gamma B_0$$

Gyromagnetic ratios of some nuclei

Definition of gyromagnetic ratio, γ:

> *The gyromagnetic ratio, γ, of a particle or system is the ratio of its magnetic dipole moment to its angular momentum*

For body of charge q, mass m rotating about an axis of symmetry:

$$\gamma = \frac{qe}{2m}$$

where e is the magnitude of the charge on the electron

For proton, $q = 1$, $m = m_p$, the proton mass.

$\gamma\!\!\!-$ is sometimes used instead of γ:

$$\gamma\!\!\!- = \frac{\gamma}{2\pi}$$

nucleus	γ (rad MHz T^{-1})	ɣ = γ / 2 π
^{1}H	267.513	42.576
^{2}H	41.065	6.536
^{3}He	203.789	32.434
^{7}Li	103.962	16.546
^{13}C	67.262	10.705
^{14}N	19.331	3.077
^{15}N	27.116	-4.316
^{17}O	36.264	5.772
^{19}F	251.662	40.053
^{23}Na	70.761	11.262
^{27}Al	69.763	11.103
^{31}P	108.291	17.235
^{57}Fe	8.681	1.382
^{63}Cu	71.118	11.319
^{67}Zn	16.767	2.669
^{129}Xe	73.997	11.777

Examples

Larmor equation: $\omega_0 = \gamma B_0 \quad \Rightarrow \quad \nu = \gamma\!\!\!/ B_0$

For hydrogen nucleus, ^{1}H, $\gamma\!\!\!/ = 42.58\,\mathrm{MHz/T}$

What is the resonance frequency for ^{1}H when:

- $B_0 = 1.5\,\mathrm{T}$?
- $B_0 = 3.0\,\mathrm{T}$?

What are the corresponding values for the energy splittings $\Delta E = h\nu$, where h is Planck's constant?

Populations in the two spin states

^{1}H in tissue in thermal equilibrium, so, partition between the populations in the two spin states follows the Boltzmann distribution:

$$\frac{N_+}{N_-} = \exp\left(-\frac{\Delta E}{k_{\mathrm{B}} T}\right)$$

where N_+ and N_- are the number of ^{1}H in $+\Delta E$ and $-\Delta E$ states respectively, k_{B} is Boltzmann's constant, and T is the temperature. For the human body, $k_{\mathrm{B}} T \approx 25.7\,\mathrm{meV}$, so:

$$\Delta E << k_{\mathrm{B}} T$$

Therefore:

$$N_- - N_+ \approx N_- \frac{\Delta E}{k_{\mathrm{B}} T} \approx N_S \frac{\Delta E}{2 k_{\mathrm{B}} T}$$

where $N_S = N_+ + N_- \approx 2N_- \approx 2N_+$ is the number of "available spins", i.e. the number of ^{1}H

Magnetisation

So:

$$\frac{N_- - N_+}{N_- + N_+} = \approx N_S \frac{\Delta E}{2k_\mathrm{B} T} = N_S \frac{\gamma h B_0}{4\pi k_\mathrm{B} T}$$

For $B_0 = 1.5\,\mathrm{T}$:

$$\begin{aligned} \frac{N_- - N_+}{N_S} &\approx \frac{42.58 \times 10^6 \times 6.6 \times 10^{-34} \times 1.5}{2 \times 1.38 \times 10^{-23} \times 300} \\ &\approx 5 \times 10^{-6} \end{aligned}$$

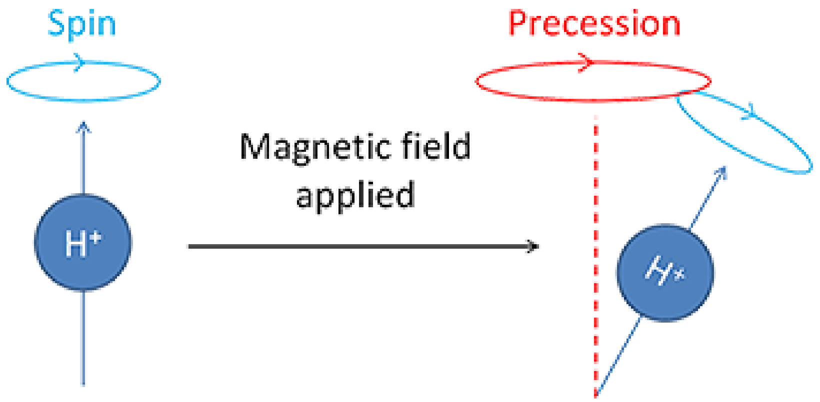
i.e. only 5 in a million protons in the body are "available" for activation in MRI at $B_0 = 1.5\,\mathrm{T}$

Bulk magnetisation is measurable

Population-density "mismatch" of ≈ 3 ppm per Tesla arises due to fact that energy splitting is small compared to $k_{\mathrm{B}} T$

Bulk magnetisation still measurable because 1 gram of water contains 10^{22} ^{1}H

Classical magnetic moment in magnetic field

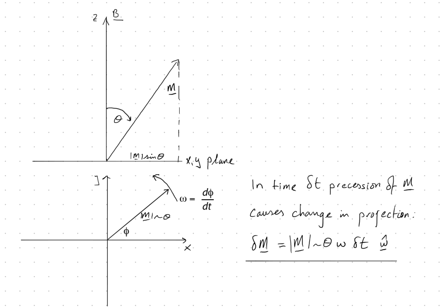


Magnetic moment that makes an angle with a magnetic field will precess around the magnetic-field axis.

Classical derivation of the Larmor equation

Classically, a magnetic moment, $\mathbf{M}$, in a magnetic field $\mathbf{B}_0$, experiences a torque given by the Bloch equation:

$$\frac{d\mathbf{M}}{dt} = \gamma \left(\mathbf{M} \times \mathbf{B}_0 \right)$$



$\mathbf{M}$ makes an angle θ w.r.t. $\mathbf{B}_0$. So:

$$\mathbf{M} \times \mathbf{B}_0 = \left(M B_0 \sin\theta \right) \hat{\boldsymbol{\omega}}_\mathbf{0}$$

So:

$$\frac{d\mathbf{M}}{dt} = \left(\gamma M B_0 \sin\theta \right) \hat{\boldsymbol{\omega}}_\mathbf{0} = \left(M \omega_0 \sin\theta \right) \hat{\boldsymbol{\omega}}_\mathbf{0}$$

Which gives the Larmor equation:

$$\omega_0 = \gamma B_0$$

Examples

Larmor equation: $\omega_0 = \gamma B_0 \quad \Rightarrow \quad \nu = \gamma\!\!\!-\, B_0$

Energy splitting: $\Delta E = \hbar\omega_0 \quad \Rightarrow \quad \Delta E = h\nu$

$h = 4.1357 \times 10^{-15}\,\mathrm{eV\,s}$

For hydrogen nucleus, $^1\mathrm{H}$, $\gamma\!\!\!- = 42.58\,\mathrm{MHz/T}$

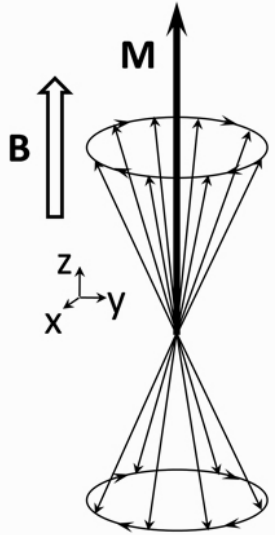
Calculating the values of ν and ΔE yields:

Magnetic field B_0 (T)	Larmor frequency (MHz)	ΔE (eV)
1.5	63.87	2.64E-07
3.0	127.74	5.28E-07

For comparison:

- FM radio waveband runs from 88.1 MHz to 108.1 MHz;
- $k_\mathrm{B} T = 2.59 \times 10^{-2}\,\mathrm{eV}$

Larmor precession



Ensemble of ^{1}H nuclei, the majority (by $\approx 3\,\text{ppm}\,\text{T}^{-1}$) orientated parallel to $\mathbf{B}_0$ precess at equilibrium around $\mathbf{B}_0$ at the Larmor angular frequency ω_0

Net magnetisation, **M**, produced is parallel to $\mathbf{B}_0$.

There is no net magnetisation in the transverse (x, y) plane; summing all contributions gives zero

Result is that there is no change in the magnitude or direction of the magnetisation vector so no RF signal is produced

Key feature of MRI: manipulate **M** so as to produce a measurable RF signal

Summary of section 2

Larmor frequency, ω_0 determined by the gyromagnetic ratio, γ, and the applied magnetic field, B_0: $\omega_0 = \gamma B_0$

In presence of B_0 at temperature T the equilibrium magnetisation of a sample of hydrogen nuclei is small, but measurable, and aligned with the applied magnetic field

Magnetisation vector, $\mathbf{M}$, created by unequal number of ^{1}H spins parallel and anti-parallel to the applied magnetic field $\mathbf{B}_0$

The magnetisation vector precesses around the direction defined by the applied magnetic field at the Larmor frequency, ω_0

The Larmor frequency is given by:

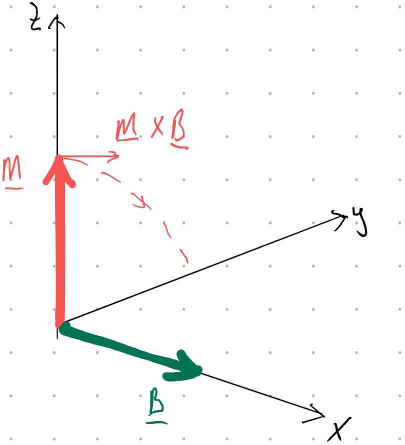
$$\omega_0 = \gamma B_0$$

This is the same Larmor frequency that was obtained in the quantum-mechanical discussion of the splitting of the energy level of the ^{1}H nucleus

Section 3

Rotating the magnetisation

First, a static example



Consider magnetisation **M** parallel to z axis and $\mathbf{B}_1$ parallel to the x axis, as shown

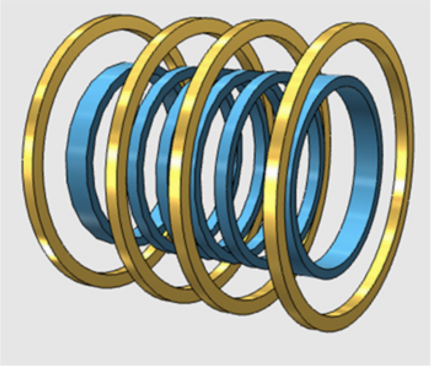
Torque, $\mathbf{M} \times \mathbf{B}_1$, is therefore parallel to the y axis

Net result is that **M** will precess around the x axis towards the y axis

This is what is done in MRI . . .

Rotating the magnetisation vector in MRI; principle

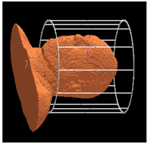
Main field, $\mathbf{B}_0$, produced with solenoid



Induces magnetisation **M** parallel to $\mathbf{B}_0$

To rotate **M** away from $\mathbf{B}_0$ require magnetic field in transverse (x, y) plane

Call the field in the x, y plane $\mathbf{B}_1$; can be produced with a variety of coil arrangements, e.g. dipole or, more efficient, a “bird cage”



To cause **M** to precess require that magnetic field oscillates at the Larmor frequency, ω_0.

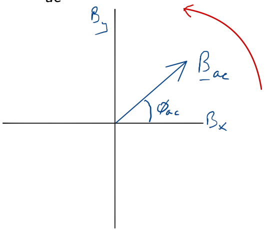
...i.e. require RF magnetic field $\mathbf{B}_1$

Rotating the magnetisation vector in MRI; mathematics

Take $\mathbf{B}_1$ to be "plane polarised" in x, y such that $B_{1_x} = B_1 \cos(\omega_0 t + \alpha)$ and $B_{1_y} = B_1 \sin(\omega_0 t + \beta)$; α and β are phases

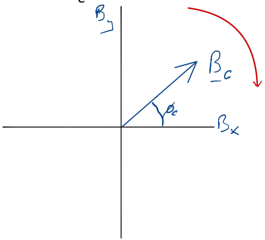
$\mathbf{B}_1$ can be rewritten in terms of two circularly polarised fields:

$\mathbf{B}_{1_{ac}}$; anti-clockwise



$$B_{1_{ac}} = \frac{B_1}{2};\ \phi_{ac} = \omega_0 t + \alpha'$$

$\mathbf{B}_{1_c}$; clockwise



$$B_{1_c} = \frac{B_1}{2};\ \phi_c = \omega_0 t + \beta'$$

Rotating the magnetisation vector in MRI

One of the two counter rotating fields will rotate in the same direction as the nuclear precession

Either $B_{1_{ac}}$ or B_{1_c} will appear stationary in the plane transverse to $\mathbf{B}_0$ in the frame that is co-rotating with the precession of the net magnetisation vector. Call the co-rotating field B_1^+

B_1^+ is equal to either $B_{1_{ac}}$ or B_{1_c} depending on the direction of $\mathbf{B}_0$

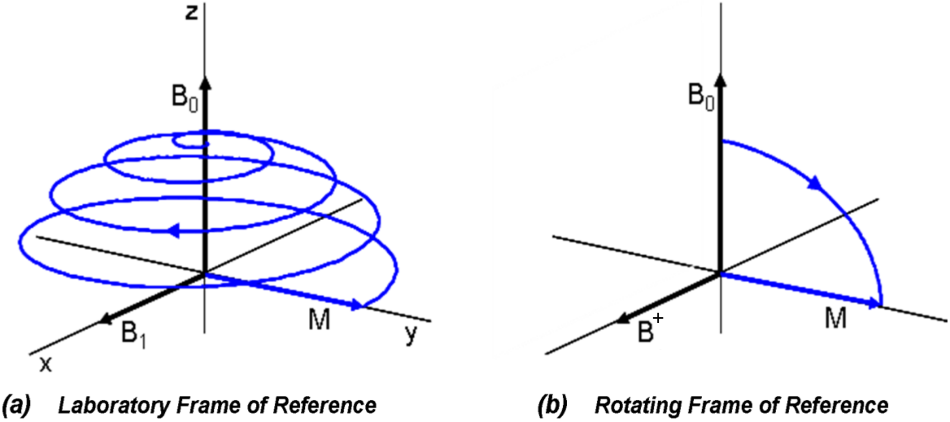
The field stationary in the rotating frame will therefore cause **M** to precess about a rotating axis in the transverse plane

The net result is that **M** can be rotated into the x, y plane where it will continue to precess around $\mathbf{B}_0$

The precession of **M** in the x, y plane gives a detectable RF signal

Rotating the magnetisation vector in MRI

M is initially parallel to $\mathbf{B}_0$



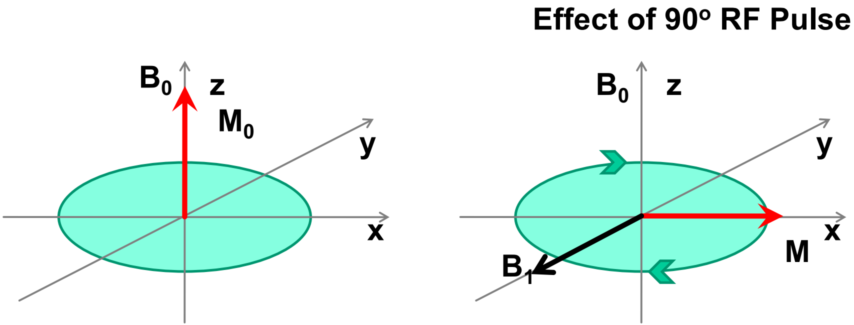
The flip angle

The flip angle, α, is proportional to the magnitude and duration of the RF pulse:

$$\alpha = \gamma B_1 t_P$$

where t_P is the duration of the RF pulse

90° pulse rotates magnetisation into transverse plane where it continues to precess



Example: calculating the duration of a 90° pulse

RF transverse magnetic field pulse is applied to rotate **M**

The magnitude of B_1 is $10\,\mu$T (i.e. 10^{-5} T)

At what rate will **M** rotate away from the $\mathbf{B}_0$ axis?

How long will it take for the flip angle to reach 90°?

Example: calculating the duration of a 90° pulse

Half an answer . . .

The magnitude of B_1 is 10 μT (i.e. 10^{-5} T)

At what rate will **M** rotate away from the $\mathbf{B}_0$ axis?

It will rotate at the Larmor frequency, f_1, arising from the field B_1, i.e. $f_1 = \not\gamma B_1$

How long will it take for the flip angle to reach 90°?

The angle can be obtained by solving the equation:

$$\frac{1}{4} = \not\gamma B_1 t_P^{90^\circ} \text{ for } t_P^{90^\circ}$$

$$\text{or } \frac{\pi}{2} = \gamma B_1 t_P^{90^\circ}$$

Summary of section 3

Net magnetisation of ^{1}H spins caused to rotate using plane-polarised, time-varying magnetic field in the x, y plane

Precession of rotated net-magnetisation vector gives rise to RF signal which can be detected

Measurement of the RF signal from the precession of rotated net-magnetisation vector is the basis of MRI